Robust Relational Contracts with Subjective Performance Evaluation *

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We study a repeated principal agent model with transferable utility, where the principal's evaluation of the agent's performance is subjective. Consequently, monitoring is noisy and private. We focus on purifiable equilibria, that are robust to small iid payoff shocks. Effort cannot be sustained in any finite memory purifiable equilibria; existing constructions fail to be purifiable. To address this problem, we allow the principal and agent to make simultaneous cheap talk announcements at the end of each period. This allows effort to be sustained with positive probability in every period, thereby we can approximate efficiency if the noise in monitoring is small. When the noise is non-negligible, we show that jointly controlled randomization that determines the agent's equilibrium effort choice can ensure payoffs arbitrarily close to full efficiency with non-vanishing noise, provided that the agents are sufficiently patient.

Keywords: Private monitoring, repeated games, jointly controlled randomization.

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1 Introduction

In many organizations, the tasks that employees must perform lack an objective measure of performance. Thus performance evaluation is subjective, and the worker cannot observe the employer's evaluation of his own performance. When coupled with moral hazard, providing incentives becomes difficult. A series of papers such – see Levin (2003), MacLeod (2003) and Fuchs (2007)) – examine the use of *relational contracts* — i.e. repeated game mechanisms – in order to provide incentives. These contracts typically require the agent to exert effort, and for the principal to pay a bonus to the worker if and only if her evaluation of the worker's performance is good. However, in order to provide incentives for truth-telling, the employer is made indifferent between paying and not paying the bonus. To do this, the equilibrium must exhibit money-burning, for example by dissolving the relationship with some probability in the event that the bonus is not paid. Since the principal is indifferent between paying the bonus or not, she has incentives to truthfully disclose her subjective evaluation of the worker's performance. However, the equilibrium exhibits an inefficiency, since productive relationships must be dissolved with positive probability. Levin (2003) and Fuchs (2007) show that efficiency can enhanced, by requiring the principal only to report on the agent's performance every T periods. As in Abreu, Milgrom, and Pearce (1991), the extent of inefficiency decreases with T, and when both players become arbitrarily patient, the one can approach full efficiency.

Our paper begins with the observation that the equilibria so constructed are fragile, and do not survive if principal and agent are subject to small iid payoff shocks. For example, the agent's cost of effort and his outside option in each period could be subject to a random shock, as could the principal's flow revenues. In this case, the equilibria where the principal is indifferent between paying the bonus or not do not survive, since the principal strictly prefers to pay the bonus when he learns that flow revenues in the next period are high, and strictly prefers not to pay it when he learns that they will be low. In consequence, she will condition her bonus payment on the shock, and not on the agent's performance. More generally, we show that in any *purifiable* finite memory equilibrium of the infinite horizon game played between the two players, the agent will not exert effort, rendering the relationship unprofitable. In particular, since all the above mentioned equilibria in the literature exhibit finite memory, none of them survive when there are payoff shocks.

This leads us to modify the game played between principal and agent, by allowing the players to make simultaneous cheap-talk announcements. We consider an equilibrium where the principal announces her private signal, while agent discloses his choice of effort (we require randomization on the part of the agent; otherwise, the agent's announcement would be redundant). Incentives for truth-telling are provided by dissolving the relationship with positive probability whenever the announcements "differ" – i.e. when the principal announces a good signal and the agent announces that he shirked, or when the principal announces a bad signal and the agent worked. We are able to construct an equilibrium where the agent works with arbitrarily high probability, and this is close to being efficient if the noise in monitoring is small. Moreover, such an equilibrium is purifiable.

When the noise in monitoring is large, we are lead to explore equilibria where announcements are made only every T periods. However, a difficulty arises, since any equilibrium with announcements requires the agent to shirk with positive probability in every period. In particular, the agent must be indifferent between always shirking and always working, and this indifference seems to preclude the efficiency gains that arise from the block structure. However, we show that this difficulty can be circumvented if the equilibrium action of the agent in the block is dictated via jointly controlled randomization. That is, the equilibrium recommendation for the action sequence for the block depends upon the realization of a random variable that is private to the agent, and a public random variable. By using such a construction, we show that as players become arbitrarily patient, there exists an equilibrium which approximates the fully efficient payoff.

1.1 Related literature

There is a large literature on repeated games with private monitoring that is relevant. Notaby, Sugaya (2013) proves a folk theorem for private monitoring games. The present paper differs from this literature in two dimensions. First, the stage game considered here has a non-trival extensive form, whereas the folk theorem obtains for simulteneous move stage games. Second, and more important is our insistence on equilibria that are robust to private payoff shocks, and are purifiable.[1]

In section 3 we show that communication allows us to overcome the impossibility result set out in proposition 1. In pioneering work in private monitoring, Compte (1998) and Kandori and Matsushima (1998) prove a folk theorem by using cheap talk announcements to coordinate behavior. We discuss the different role that cheap talk plays in our context at the end of 3

Finally, jointly controlled randomization plays a role similar to that of the mediator in

[1]Bhaskar, Mailath, and Morris (2008) examine the purifiability of belief-free strategies in the (simultaneous move) prisoners' dilemma.

Rahman (2012).

2 The basic model

Time is discrete and the horizon infinite. In each period, the agent chooses from $\{E, S\}$, where the cost of effort E is $c > 0$, while the cost of shirking is zero. Output y is stochastic, and takes values in the set $\{G, B\}$.[2] Assume $1 > Pr(y = G|E) =: p > Pr(y = G|S) =: q > 0$. Let $\bar{y}$ denote the expected value of output when E is chosen, let $-\ell$ denote expected output when S is chosen. Assume that $\bar{y} - c > 0 > -\ell$, and normalize the outside options of the two parties to zero. Thus it is efficient for the agent to be employed and to choose effort, but if the agent shirks, then it is preferable to dissolve the relationship. Both parties are risk-neutral and there do not face limited liability constraints. They maximize the discounted sum of payoffs, with common discount factor δ.

In the interests of precision, let us consider the following stage game Γ that is played in every period, conditional on the relationship not having been terminated.

- The agent is paid a base wage w and chooses $a \in \{E, S\}$.
- The principal observes $y \in \{G, B\}$ and decides whether to pay a bonus or not, over and above the base wage w.
- Principal and agent observe the realization of a public randomization device and simultaneously decide whether to terminate the relationship or not – the relationship continues to the next period if and only if both parties want to continue.

We denote the game that is repeated infinitely often, conditional on non-termination, by Γ^∞.

The fundamental problem is that monitoring is imperfect and private. The principal does not observe the agent's action, and the agent does not observe y.To incentivize effort, the agent's bonus payments (or his continuation value) must depend upon the principal's observation of output. However, since this observation is private, the principal's continuation

[2] Our analysis extends to any finite signal space Y. Let B denote the signal that has the largest likelihood ratio, i.e. B denotes the signal y that maximizes $\frac{Pr(y|S)}{Pr(y|E)}$. When δ is large, efficiency requires punishments only after the signal with the largest likelihood ratio,[3] and so we may, without loss of generality focus on a binary partition of the signal space, $\{G, B\}$, where $G = Y \setminus \{B\}$. The only complication is that in some of our arguments, we will have to evaluate incentives when the principal sees the least favorable signal in G, i.e. the one with the highest likelihood ratio.

value, net of the cost of the bonus payment, cannot depend upon the signal that the principal observes. The solution to this problem, proposed by MacLeod (2003) and Levin (2003), is to ensure that the principal is indifferent between paying the bonus, or not paying it. This can be achieved via a public randomization device that decrees that the relationship be dissolved with some probability, whenever the bonus is not paid. In other words, a part of the surplus from the relationship must be destroyed, since the agent cannot be punished while simultaneously rewarding the principal.

Two problems arise with this repeated game equilibrium/relational contract. First, it is inefficient, since some surplus is destroyed. Levin (2003) and Fuchs (2007) show that inefficiency can be mitigated if the players are patient, by dividing the interaction into blocks of T periods. The bonus is withheld only if the agent fails in every period in the block, and this reduces the loss in surplus.

As in the one period construction, the equilibrium relies on the principal's indifference between paying the bonus and not paying it, and on her breaking this indifference according to the history of private signals. Consequently it is fragile. In particular, if the value of the relationship to the principal is is subject to small shocks that are privately observed by the principal, then she will condition her bonus payment on the realization of these shocks, and not upon output signals.

We now make this argument more precise. The perturbed version of the stage game, $\Gamma(\xi)$, is as follows:

- The agent observes a random shock z_1 before he chooses his action, and his cost of effort is augmented by ξz_1.

- The principal observes a shock z_2 that affects her flow payoff from the relationship for the next period, before she makes the bonus decision, that is, thd value of the output equals $y + \xi z_2$.

- The agent observes a random shock z_3 that augments his outside option by ξz_3 , before the quitting decision.

The shocks z_i are independently distributed and each has an atomless distribution. We denote the repeated perturbed game by $\Gamma^\infty(\xi)$.

An equilibrium σ of the repeated game Γ^∞ is *purifiable* if for any sequence $\xi \to 0$, there exists a sequence of equilibria $\sigma(\xi)$ of $\Gamma^\infty(\xi)$ such that the associated behavior converges to σ.

Proposition 1 *Let σ be a purifiable finite memory equilibrium of the unperturbed game. In any such equilibrium, the worker always shirks when hired; the principal never pays a bonus, and the agent always quits and the principal terminates the relation*

Proof.

See appendix. ■

The idea of the proof is similar to that set out in Bhaskar, Mailath, and Morris (2013). It does not follow immediately from that proof for three reasons. First, in the present game, the principal has a continuous action space (of bonus payments), and the perturbations are lower dimensional. Second, the termination decisions are taken simultaneously, whereas the earlier result relies on sequential moves.

In particular, this proposition implies that the equlbria considered in work of MacLeod (2003), Levin (2003) and Fuchs (2007) are not purifiable, being of finite memory.

A similar problem arises when we consider other ways of burning money. For example, it is suggested that one can provide incentives for truth-telling by the principal by making her indifferent between paying the bonus and contributing to charity. Once again, incentive compatibility is destroyed by the slighest deviation of the principal's preferences from exact indifference. For example, in each period, the principal may have a slight privately known preference for contributing to charity rather than paying the worker, or vice-versa. It can be shown that proposition 1 would apply in this version of the model as well.

3 Cheap talk

We modify the stage game Γ set out in the previous section, by allowing principal and agent to make simultaneous cheap talk announcements. Specifically, after the agent has chosen effort, and the principal has observed output, there is a cheap talk stage where the parties announce their private information. The equilibrium we construct requires that principal reports the signal she observed, i.e. either g or b, while the agent reports his action choice, i.e. he reports e or s. If the principal reports g, then the equilibrium requires that he pay a bonus B.

Our equilibrium requires the agent to choose both actions with positive probability, so that he chooses E with probability π. Equilibrium requires "truth-telling" at the cheap talk stage. If the reports "coincide", i.e. are either (e, g) or (s, b), then the relationship continues to the next period. If the reports differ, then the relationship is terminated with positive probability, depending upon the realization of a public randomization device. Termination occurs with probability α if the reports are (e, b) and with probability $\kappa\alpha$ if the reports are (s, g). Let V^P and V^A denote the values of the relationship, to principal and agent respectively.

We choose the bonus B so that it exactly incentivizes effort, i.e. it must satisfy:

$$c = (p - q)\,B. \tag{1}$$

Given that the agent is compensated for effort via the bonus, he is indifferent between exerting effort or not when the probability of termination is equal after both actions, i.e. κ satisfies:

$$\kappa = \frac{1-p}{q}. \tag{2}$$

Given the expected bonus B, and the above value of κ, the agent is indifferent between E and S and thus willing to randomize. We turn to truth-telling incentives at the cheap talk stage.

If the agent has chosen E, then his payoff loss from announcing e arises when the principal sees a bad signal, and equals $(1-p)\alpha V^A$. His payoff loss from announcing s is $p\kappa\alpha V^A$. Thus truth-telling after choosing E is optimal if

$$\kappa = \frac{1-p}{q} \geq \frac{1-p}{p}, \tag{3}$$

which is satisfied since $q < p$. On the other hand, if the agent has chosen S, then his payoff loss from announcing s is $q\kappa\alpha V^A$, and from announcing e is $(1-q)\alpha V^A$, and thus truth telling at S is optimal if

$$\kappa = \frac{1-p}{q} \leq \frac{1-q}{q}, \tag{4}$$

which also follows from the same inequality $q < p$.

We turn now to truth telling incentives for the principal. If the principal reports b at B, her payoff loss is

$$L(b|B) = \frac{\pi(1-p)}{\pi(1-p) + (1-\pi)(1-q)}\alpha V^P. \tag{5}$$

Her expected loss from announcing g is

$$L(g|B) = (1-\delta)B + \frac{(1-\pi)(1-q)}{\pi(1-p) + (1-\pi)(1-q)}\kappa\alpha V^P. \tag{6}$$

Since the loss from the announcing b should be smaller, we get the condition:

$$\pi(1-p)[\alpha V^P - (1-\delta)B] \leq (1-\pi)(1-q)[\kappa\alpha V^P + (1-\delta)B]. \tag{7}$$

Thus the incentive constraint for the principal at B reduces to

$$\frac{\pi}{1-\pi} \leq \left(\frac{1-q}{(1-p)}\right)\frac{\kappa\alpha V^P + (1-\delta)B}{\alpha V^P - (1-\delta)B}. \tag{8}$$

Second, the principal should be willing to announce g when she sees G. Her loss from announcing g at G is

$$L(g|G) = (1-\delta)B + \frac{(1-\pi)q}{\pi(1-(1-p)) + (1-\pi)q)}\kappa\alpha V^P. \tag{9}$$

Her loss from reporting b at G equals

$$L(b|G) = \frac{\pi(p)}{\pi(p) + (1-\pi)q}\alpha V^P. \tag{10}$$

Thus the incentive constraint for the principal at G is:

$$\frac{\pi}{1-\pi} \geq \left(\frac{q}{p}\right)\frac{\kappa\alpha V^P + (1-\delta)B}{\alpha V^P - (1-\delta)B}. \tag{11}$$

An equilibrium consists of a triple (π, α, V^P) – a mixing probability for the agent, a termination probability after (e, s) and the principal's value – such that the principal's truth-telling constraints 11 and 8 are satisfied, and the V^P is indeed the value generated by the equilibrium:

$$V^P(\alpha, \pi) = (1-\delta)\left(\pi(\bar{y} - c + \ell) - w - qB - \ell\right) + \delta\left(1 - \alpha(1-p)\right)V^P. \tag{12}$$

. The agent's value in such an equilibrium equals

$$V^A(\alpha, \pi) = (1-\delta)(w + qB + \delta\left(1 - \alpha(1-p)\right)V^A. \tag{13}$$

The total value equals $V(\alpha, \pi) = V^P(\alpha, \pi) + V^A(\alpha, \pi)$.

Observe from 8 that we must have

$$\alpha V^P \geq (1-\delta) \geq \frac{(1-\delta)c}{p-q},$$

since the denominator the right-hand side must be positive. Furthermore, since V^P is increasing in π, and 3 must be satisfied, we have the following upper bound $\bar{V}^P$ on the principal's value:

$$\bar{V}^P = (\bar{y} - c - w - qB) - \delta \frac{(1-p)c}{p-q}. \tag{14}$$

Let the corresponding value of α be $\bar{\alpha}$. Thus $\bar{\alpha}\bar{V}^P = (1-\delta)B$.

The consequent upper bound on the total value is given by

$$\bar{V} = (\bar{y} - c) - \delta \frac{(1-p)c}{p-q}. \tag{15}$$

Proposition 2 *Let $\Delta > 0$. There exists a purifiable equilibrium with agent mixing probability $\pi < 1$ and termination probability $\alpha > \bar{\alpha}$ which generates a total value $V(\alpha, \pi)$ such that $|\bar{V} - V(\alpha, \pi)| < \Delta$.*

Proof. It suffices to show that $|\bar{V}^P - V^P(\alpha, \pi)| < \Delta$ for arbitrary Δ. Since $V^P(\alpha, \pi)$, as defined by equation 12, is a differentiable function, it is straightforward to verify that it is increasing in π and decreasing in α (as long as V^P is positive). Furthermore, $\alpha V^P(\alpha, \pi)$ is increasing in α, and thus at any (α, π) where the incentive constraint 3 binds, it is possible to increase α and make it hold strictly.

Consider $\alpha = \bar{\alpha}, \pi = 1$. Consider a small increase in α above $\bar{\alpha}$. The right-hand sides of 11 and 8 are now finite and different from each other, so that it is possible to find a value of $\pi < 1$ that satisfies these two incentive constraints. By choosing α sufficiently close to $\bar{\alpha}$, the difference $\alpha V^P(1, \alpha) - (1-\delta)B$ can be made sufficiently small, so that π can be made as close to one as required, since the right-hand side of 8 converges to infinity as $\alpha V^P(1, \alpha) - (1-\delta)B \downarrow 0$. Since $V^P(.)$ is continuous, this suffices to prove the theorem. ■

It remains to show that this equilibrium is purifiable. Both players have strict incentives at the cheap talk stage. Also, when the relationship is to be terminated, this is common knowledge, and we can provide strict incentives to terminate for both parties by specifying continuation play where the agent always shirks and the bonus is never paid. The only remaining question is showing that the agent's random choice of effort can be purified, and this raises no problems.

Observe that to approximate full efficiency, one needs that $p \to 1$, i.e. vanishing noise in monitoring.

In their pioneering work on repeated games with private monitoring, Compte (1998) and Kandori and Matsushima (1998) prove a folk theorem by using cheap talk announcements to coordinate behavior. In the case where signals are independent conditional on the action profile, the equilibria they construct have a "belief-free" flavor, since each player is made indifferent between her possible announcements. Cheap-talk plays a different role here, since it is a way for providing strict incentives for truth-telling. Furthermore, randomization by the agent also plays an essential role in providing strict incentives, whereas randomization plays no such role in this earlier work.

4 T-period equilibrium with cheap talk?

We now examine whether the basic construction, for the one-period case, can be extended to T periods, in the usual block manner.

Recall the one-period construction. Consider the non-purifiable pure strategy equiibrium, where the bonus compensates the worker for effort, and where the boss is made indifferent between paying the bonus and not, by choosing the termination probability α. We showed that with cheap talk, this equilibrium can be approximated by an equilibrium with the following features:

- The worker mixes between E and S.
- The bonus compensates the worker for the cost of effort.
- Both players have strict incentives at the cheap talk stage.

Let us now examine the conditions that must be satisfied in a $T-$ period construction. Suppose that the equilibrium requires the agent to choose E with high probability in every period of the block. Then, the play of E is any period k of the block must be incentivized, so that the principal's reporting decision must depend on whether signal G or B is realized in that period. However, if the principal is to have strict reporting incentives, then the $k-$th period signal must be informative of the agent's behavior (and hence his report). This is only possible if the agent chooses both E and S with positive probability in the $k-$th period of the block. In other words, both E and S must be played with positive probability in each period of the block.

If we want the sequence $E, E, .., E$ to be played with high probability, and if we also require S to be played with positive probability in every period, then the efficient way to achieve both requirements is for the sequence $S, S, ..., S$ to be also played with positive probability. In other words, the agent must be indifferent between the two sequences – "always E" and "always S" – at the beginning of the block.

Now the cheapest way to make the agent indifferent between the two sequences is to punish the agent if and only if all signals in the block are B. However, if indifference between the two effort sequences is achieved in this manner, then the agent strictly prefers to choose E in the first period, and S in subsequent periods, to either of these alternatives. Conversely, if the punishment after the signal realization of "all B" is increased, so as to deter this deviation, then the agent will find "always S" to be inferior, and will not randomize in the required manner.

In other words, if the agent is to be made indifferent between these two sequences, and also deterred from deviating to other action sequences, then the agent must also be punished after intermediate signal realizations – i.e. realizations where some signals are good while the others are bad. However, this directly reduces the efficiency of equilibria, since the role of the block structure is to reduce the likelihood of punishments when the agent chooses "always E".

These heuristic arguments can be made precise for the case of $T = 2$. In particular, we show that in any equilibrium where the agent is indifferent between EE and SS, the efficiency loss associated with the equilibrium is no less than that associated with the best equilibrium with $T = 1$, i.e. where punishments are contingent only on signal realizations within the period.

In order to make this argument, we can abstract from considerations of purification. As in Chan and Zheng (2011), let us consider the mechanism design problem, where at the end of two periods, the principal must decide whether to pay the agent or burn money depending on the signal realizations. Our focus is on the minimal amount of money that must be burned, in order to incentivize the agent, so that she is indifferent between EE and SS, and so that both these sequences are optimal.[4] Let Z_{ij} denote the amount of money, in period one payoff, that is burned after signal $i \in \{G, B\}$ is realized in period 1 and $j \in \{G, B\}$ is realized in period 2. Thus, the problem is to minimize

$$p^2 Z_{GG} + p(1-p)[Z_{GB} + Z_{BG}] + (1-p)^2 Z_{BB}, \tag{16}$$

subject to the condition that the agent be indifferent between EE and SS:

$$(2-p-q)Z_{BB} - (p+q)Z_{GG} + (p+q-1)[Z_{GB} + Z_{BG}] = \frac{(1+\delta)c}{p-q}. \tag{17}$$

In addition the following incentive constraints need to be satisfied. First, the agent must

[4] In Chan and Zheng (2011) it suffices to ensure that EE is optimal.

prefer EE to SE:

$$p[Z_{BG} - Z_{GG}] + (1-p)[Z_{BB} - Z_{GB}] \geq \frac{c}{p-q}, \tag{18}$$

Observe, that if the above condition is satisfied, then the indifference condition 17 implies that the prefers SS to ES.

Second, the agent must prefer EE to ES:

$$p[Z_{GB} - Z_{GG}] + (1-p)[Z_{BB} - Z_{BG}] \geq \frac{\delta c}{p-q}. \tag{19}$$

Once again, if this condition is satisfied, the agent automatically prefers SS to SE, due to the indifference condition. Thus the problem is to minimize 16 subject to the equality constraint 17 and the inequality constraints 18 and 19, and the non-negativity constraints on the choice variables, Z_{ij}. Since this program is linear and increasing in the choice variables, the Z_{ij}, we may set $Z_{GG} = 0$ – if there is a solution to the problem with $Z_{GG} = 0$, then increasing Z_{GG} will merely increase the cost.

The following observations are useful in deriving the solution heuristically, without recourse to too much algebra. First, observe that if one were to minimize 16 subject only to the indifference condition 17, then one would choose $Z_{GG} = Z_{BG} = Z_{GB} = 0$, i.e. incentives would be provided solely through Z_{BB}. More generally, one would like to minimal values of the sum $Z_{GB} + Z_{BG}$.

It is useful to write down the constraint that the agent must prefer SS to ES explicitly:

$$q[Z_{BG} - Z_{GG}] + (1-q)[Z_{BB} - Z_{GB}] \leq \frac{c}{p-q}. \tag{20}$$

Observe that by subtracting 20 from 18, one derives the following necessary condition:

$$[Z_{BG} + Z_{GB}] - [Z_{BB} + Z_{GG}] \geq 0. \tag{21}$$

In other words, the first inequality constraint, in conjunction with the indifference condition, provides a lower bound on the sum $Z_{GB} + Z_{BG}$, which is given by Z_{BB} (given that $Z_{GG} = 0$). Now if we set $Z_{GB} + Z_{BG} = Z_{BB}$, then the value of the objective simplifies to $(1-p)Z_{BB}$, while the indifference condition 17 implies that

$$Z_{BB} = \frac{(1+\delta)c}{p-q}. \tag{22}$$

Consequently, the efficiency loss associated with the equilibrium is

$$\frac{(1-p)(1+\delta)c}{p-q}, \tag{23}$$

which is equal to the loss sustained via one-period strategies. In other words, *there is no efficiency gain by using a two-period block.*

The formal proof that the above is indeed the optimal solution to the linear program is as follows. First, we argue that both inequality constraints must bind at the solution. If the constraint 19 does not bind, this implies $Z_{GB} = 0$; however, the necessary condition 21 then implies that the constraint 19 is violated. Consequently, we have three equations 17, 18 and 19 that uniquely determine the values Z_{BB}, Z_{BG} and Z_{GB}. These are consistent with the heuristic solution above.

5 Secret instructions

We now show that the problem can be remedied, by letting the principal provide *secret instructions* to the agent, where the instruction is hidden from the instructor. The agent is required to choose one the two sequences "always E" and "always S", where the choice is dictated by a randomization device that is not under his control. Consequently, the agent does not have to be indifferent between these two sequences, even though the equilibrium requires him to play both with positive probability. Specifically, at the beginning of the T period block, the following sequence of events unfolds:

- The agent records the realization of private random variable x that is uniformly distributed on $[0, 1]$.
- The principal observes the realization of a public random variable y that is uniform on $[0, 1]$, and announces this to the agent.
- Let $z :=$ mantissa $(x + y)$.[5] Note that $z|x$ is uniform on the unit interval for every value of x (and so is $z|y$ for every value of y).
- If $z \leq \pi$, the agent chooses "always E"; otherwise, he chooses "always S".
- At the end of the block, the principal reports whether every signal was bad, i.e. report b or at least one signal was good, report g.

[5] That is, $z = x + y$ if $z + y < 1$, and $z = x + y - 1$ otherwise.

- The agent's record of his private random variable x is revealed, thereby revealing the secret instruction z. If the principal reports g, and the agent was required to play E, the principal pays a bonus B to the agent; otherwise, he does not.

- The relationship is terminated with some probability after: $(E, b), (S, g)$, and continues for sure after (E, g) or (S, b).

The critical difference as compared to cheap talk is that the agent does not have to be indifferent between always playing E and always playing S. Indeed, the efficient equilibrium has his incentive constraint after the recommendation to play E holding with equality, so that he is indifferent between working and shirking in the first period, but strictly prefers to work thereafter.[6] Consequently, it should be possible to approximate full efficiency as $\delta \to 1$, by choosing T sufficiently large.

Let us first verify the agent's incentives. If z recommends S, then choosing effort in any period does not result in any bonus payment, and only increases the probability of termination, by increasing the probability that at least one G is observed. To verify incentives after the recommendation E, let us set the bonus so that it exactly compensates the agent for his first period effort, given that he has been recommended to play E. That is the discounted expected loss of bonus equals the cost of the effort in the first period of the block.

$$B = \frac{c}{(p-q)(1-p)^{T-1}\delta^T}. \tag{24}$$

Since the agent also loses $\alpha V^A > 0$ after (e, b), this implies that his first period incentive constraint holds strictly, for any $V^A > 0$. Furthermore, by the standard Abreu, Milgrom, and Pearce (1991) argument, if the randomization device recommends E, , then he does not have an incentive to shirk in any other period of the block, independent of his own actions.

We also choose κ as before, so that the probability of termination is the same after either recommended action of the randomization device – this is no longer necessary, but simplifies calculations.

For the principal, we have the necessary condition:

$$\alpha V^P \geq (1-\delta)B = \frac{(1-\delta)c}{(p-q)(1-p)^{T-1}\delta^T}, \tag{25}$$

[6] In fact, we can choose to make this incentive constraint hold as a strict inequality, thereby ensuring strict incentives for the agent at every information set.

which is satisfied, for any given T, if δ is large enough. The remaining condition is verifying truth-telling for the principal.

The incentive constraint for the principal at "all B" reduces to

$$\frac{\pi}{1-\pi} \leq \left(\frac{1-q}{(1-p)}\right)^{T} \frac{\kappa q V^{P}}{qV^{P}-(1-\delta)B}. \tag{26}$$

The incentive constraint for the principal when she observes a single G, that she is willing to announce g, is

$$\frac{\pi}{1-\pi} \geq \left(\frac{(1-q}{(1-p)}\right)^{T-1} \left(\frac{q}{p}\right) \frac{\kappa \alpha V^{P}}{\alpha V^{P}-(1-\delta)B} \tag{27}$$

Since the right-hand side of 26 is strictly greater than that of 27, it is possible to find π such that both incentive constraints are satisfied. Further, by taking α sufficiently small, we can ensure that the right hand side of 26 can be made arbitrarily large, and so π can be chosen close to one.

The principal's payoff, when α is chosen to be minimal, is given by:

$$V^{P} \leq (\pi(y+\ell-c)-w-\ell) - \frac{(1-p)c}{(q-p)(1+\delta+..+\delta^{T-1})}. \tag{28}$$

Since π can be made arbitrarily close to one for any T, and since T can be chosen to be arbitrarily large when δ is large enough, we can approximate the efficient payoff as $\delta \rightarrow 1$.

Observe that the equilibria we have constructed are sequentially strict – each player has strict incentives at each information set. Furthermore, for a given T, there are finitely many strategically distinct information sets in the game. Consequently, the equilibrium is purifiable.[7] We therefore have the following proposition:

Proposition 3 *In the repeated game with a secret instructions, there exist purifiable equilibria that can approximate the fully efficient payoff provided that δ is sufficiently close to one.*

6 Concluding Comments

We. have focused on extending the basic interaction between principal and agent, either by allowing for cheap talk or secret instructions. Alternatively, if we allow for the agent's effort

[7]See Bhaskar, Mailath, and Morris (2013).

to have persistent effects on output, then one can construct purifiable equilibria.

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